Antarctica J. Math., 9(6)(2012), 503-511
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NEURAL NETWORK TECHNIQUE TO ESTIMATE THE SOLUTIONS OF BURGERS-HUXLEY AND BURGERS-FISHER EQUATIONS

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(Received on 2nd May 2012)

In this paper, we propose an artificial neural network technique to solve the well-known partial differential equations of the types: Burgers–Huxley and Burgers–Fisher. The results obtained by this technique were very accurate, simple and convenient. Moreover we compared the approximated solutions with the exact one, we found them in good agreement with each other, and this is because of superior properties of Neural Network, i.e. parallel in the domain by less computational cost.

MSC 2000: 35Q53, 37K10
Keyword: Burgers-Fisher Equation, Burgers-Huxley Equation, Nonlinear PDE, and Artificial Neural Network.

1. INTRODUCTION

The Burgers-Fisher equation has a wide range of applications in plasma capillary-gravity waves, nonlinear optics and chemical physics. This equation shows a prototypical model for describing the interaction among the reaction mechanism, convection effect, and diffusion transport [1].

Many researchers have spent a great deal of effort to compute the solution of the Burgers-Fisher equation using various numerical methods. Recently, Kocacoban *et al* [2] presented Reduced Differential Transformation Method to solve Burgers-Fisher Equation, In [3] a Burgers-Fisher equation is solved by using the Adomian's Decomposition Method (ADM), Modified Adomian's Decomposition Method (MADM), Variational Iteration Method (VIM), Modified Variational Iteration Method (MVIM), Modified Homotopy Perturbation Method (MHPM) and Homotopy Analysis Method (HAM).

Generalized Burgers–Huxley equation plays an important role in mathematical physics. In a recent year, some work had been done in order to find the numerical solution to this equation, for example see [4, 5]. Adomian's Decomposition Method for solutions of Burgers-Huxley and Burgers-Fisher equations were obtained by Ismail *et al*. [6].

In [7] the Homotopy analysis method (HAM) is applied to obtain approximate analytical solution of the generalized Burgers-Huxley.

2. THE MODEL PROBLEMS

The analysis presented in this paper is based on the generalized Burgers-Fisher equation:

$$u_t + \alpha u^{\delta} u_x - u_{xx} = \beta u(1-u^{\delta}),\ 0 \le x \le 1,\ t \ge 0, \tag{1}$$

with the initial condition: $u(x,0) = f(x)$.

The exact solution of the above equation is:

$$u(x,t) = \left[\left(\frac{1}{2}+\frac{1}{2}\right)\tanh\left[\frac{-\alpha\delta}{2(\delta+1)}\left(x-\left(\frac{\alpha}{(\delta+1)}+\frac{\beta(\delta+1)}{\alpha}\right)t\right)\right]\right]^{\frac{1}{\delta}},$$

while the solution of the generalized nonlinear Burgers-Huxley equation is:

$$u_t + \alpha u^{\delta} u_x - u_{xx} = \beta u(1-u^{\delta})(u^{\delta}-\gamma) \quad , \forall \quad 0 \le x \le 1\ ,\ t \ge 0\ , \tag{2}$$

with the initial condition **:** $u(x,0)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh(A_1 x)\right)^{\frac{1}{\delta}}$.

In [8], it was shown that: $u(x,t)=\left(\frac{\gamma}{2}+\frac{\gamma}{2}\tanh\left[A_1(x-A_2 t)\right]\right)^{\frac{1}{\delta}}$,

where:

$$A_1=\frac{-\alpha\delta+\delta\sqrt{\alpha^2+4\beta(1+\delta)}}{4(1+\delta)} ,$$

$$A_2=\frac{\gamma\ \alpha}{(1+\delta)}-\frac{(1+\delta-\gamma)\left(-\alpha+\sqrt{\alpha^2+4\beta(1+\delta)}\right)}{2(1+\delta)} ,$$

and α, β, γ, δ, are parameters, and $\beta\geq 0$, $\delta>0$, $\gamma\in(0,1)$.

In this paper we apply an artificial neural network to estimate the solutions of the generalized Burgers-Fisher equation (1) and the generalized Burgers-Huxley equation (2).

3. ARTIFICIAL NEURAL NETWORK (ANN)

Neural networks are computational models of the biological brain. Like the brain, a neural network comprises a large number of interconnected neurons. Each neuron is capable of performing only simple computation ([9], [10]). Anyhow, the architecture of an artificial neuron is simpler than a biological neuron. ANNs are constructed in layer connected to one or more hidden layers where the factual processing is performance through weighted connections. Each neuron in the hidden layer joins to all neurons in the output layer. The results of the processing are acquired from the output layer. Learning in ANNs is achieved through particular training algorithms which are expanded in accordance with the learning laws, assumed to simulate the learning mechanisms of biological system [11]. However, as an assembly of neurons, a neural network can learn to perform complex tasks including pattern recognition, system identification, trend prediction, function approximation, and process control [10]. Multi-layer Perceptron (MLPs) are perhaps the most common type of feed forward networks [12]. Their application in function approximation is well known [13]. Fig.1 shows an MLP which has three layers: an input layer, an output layer and a hidden layer. Neurons in input layer only act as buffers for distributing the input signals x_i to neurons in the hidden layer. Each neuron j (Fig. 2) in the hidden layer sums up its input signals x_i after weighting them with the strengths of the respective connections w_{ij} from the input layer and adding the bias b_i to them, and computes its output η_i as a function g of the sum

$$\eta_i = g(\sum w_{ij} x_i + b_i) \qquad (3)$$

where η_i is each neuron output and g can be a simple threshold function or a sigmoid.

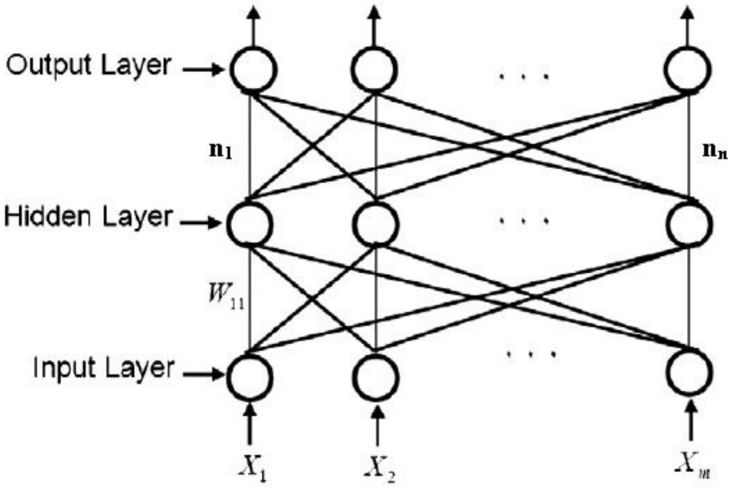

Fig.1 Neural Network

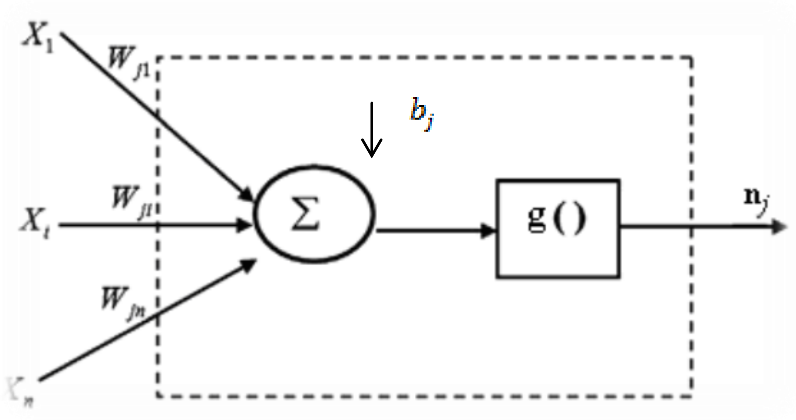

Fig.2 Details of a neuron

4. ALGORITHM

1. Create an architecture consists of two input nodes in the input layer, two hidden nodes in two hidden layers, one output node in the output layer. Assign the nodes to each layer.
2. Initialize the weights and bias to random values.

3. Initialize the network parameters.
4. Train the network with initialized parameters, and with sigmoid activation function.
5. Repeat the process until the maximum epochs are reached or the desired output is identified or the minimum gradient is reached.

EXAMPLE 1. To show the procedure, we will examine Burgers-Fisher equation (1), for computing work:

(i) We have taken $\delta = 1,\ \alpha = 0.001 \mathit{and}\ \ \beta = 0.001$, and make comparison between the exact solution and the artificial neural network solution will be given in the following (Table 1):

Table 1: Comparison of results for the solution of example 1 (i): with $\delta = 1,\ \ \alpha = \beta = 0.001$

	BURGERS-FISHER EQUATION			
	x	t	ANN Solution	Exact Solution
Train	0.1	0.02	0.499992376070278	0.499992501250001
		0.04	0.499997390931330	0.499997502500000
		0.06	0.500002618624028	0.500002503750000
		0.08	0.500004323675766	0.500007504999999
		0.1	0.500008362566994	0.500012506249997
Test	0.5	0.02	0.499942357170334	0.499942501250253
		0.04	0.499947300591024	0.499947502500193
		0.06	0.499952472907354	0.499952503750143
		0.08	0.499957827274413	0.499957505000102
		0.1	0.499963309188654	0.499962506250070
Train	0.9	0.02	0.499894669813664	0.499892501251656
		0.04	0.499899646978644	0.499897502501436
		0.06	0.499904844236115	0.499902503751236
		0.08	0.499912944578735	0.499907505001055
		0.1	0.499940151437797	0.499912506250893

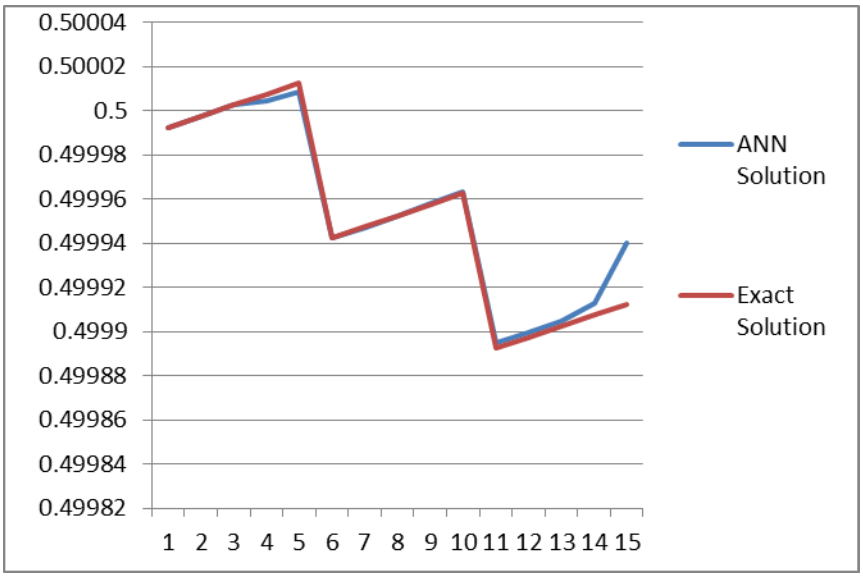

Fig3. Neural Network estimated solution for Example (i)

(ii) The Neural Network results of the generalized Burgers-Fisher equation with $\alpha = \beta = 0.001 and \quad \delta = 2$ are listed in the following (Table 2). A comparison has been made between the computed results and the exact solution.

Table 2. The ANN Solution for $\alpha = \beta = 0.001 and \quad \delta = 2$

	t	x	ANN Solution	Exact Solution
Train	0.01	0.02	0.707107967140373	0.707107960089704
		0.04	0.707105710173388	0.707105603067101
		0.06	0.707103439921334	0.707103246036642
		0.08	0.707101156763832	0.707100888998326
Test	0.04	0.02	0.707118306490439	0.707118567772678
		0.04	0.707115827729759	0.707116210785435
		0.06	0.707113382576982	0.707113853790336
		0.08	0.707110969188292	0.70711149678738
Train	0.08	0.02	0.707126113698523	0.707132711102411
		0.04	0.707125041288331	0.70713035416232
		0.06	0.707123964828714	0.707127997214372
		0.08	0.707112985753679	0.707125640258565

EXAMPLE 2. (i) The second Example is the Burgers-Huxley equation (2), for computational work, we have taken $\delta = 1, \alpha = 1 and \ \beta = 1, \gamma = 0.001$, will be given in Table 3.

Table 3. Comparison of results for the solution of example1
$\delta = 1, \ \alpha = 1 \ \ and \, \beta = 1, \ \ \gamma = 0.001$

	BURGERS-HUXLEY EQUATION			
	x	t	ANN Solution	Exact Solution
Train	0.1	0.05	0.000500018767312007	0.0005000187437499913
		0.06	0.000500019988952383	0.0005000199924999893
		0.07	0.000500021216648639	0.0005000212412499872
		0.08	0.000500022450026275	0.0005000224899999849
Test	0.5	0.05	0.0005000227908201935	0.0005000687437495669
		0.06	0.000500023947363785	0.0005000699924995427
		0.07	0.0005000251128579881	0.0005000712412495179
		0.08	0.0005000262871170913	0.0005000724899994921
Train	0.9	0.05	0.000500018676234666	0.0005001187437477677
		0.06	0.000500019933811789	0.0005001199924976965
		0.07	0.000500021198803991	0.0005001212412476238
		0.08	0.000500022470680193	0.0005001224899975496

(ii) As seen in Table 4, the solution has also been considered and seen to be very accurate.

Table 4. The ANN Solution for $\alpha = \beta = 1$, $\gamma = 0.001 \ \ and \, \delta = 2$

	X	t	ANN solution	Exact solution
Train	0.1	0.05	0.022361617389490	0.022361481349192
		0.1	0.022361793987127	0.022361797408374
		0.5	0.022364326020970	0.022364325720927
		1	0.022367531067443	0.022367485709173
		1.5	0.022370643687393	0.022370645249964
Test	0.5	0.05	0.022363110175765	0.022363423258460
		0.1	0.022363474165645	0.022363739290179
		0.5	0.022366436536923	0.022366267382955
		1	0.022369098130919	0.022369427096307
		1.5	0.022373020192181	0.022372586362010
Train	0.9	0.05	0.022365363650360	0.022365364998953
		0.1	0.022365727486832	0.022365681003197
		0.5	0.022367940373198	0.022368208876102
		1	0.022371368966615	0.022371368314439
		1.5	0.022374526253143	0.022374527304935

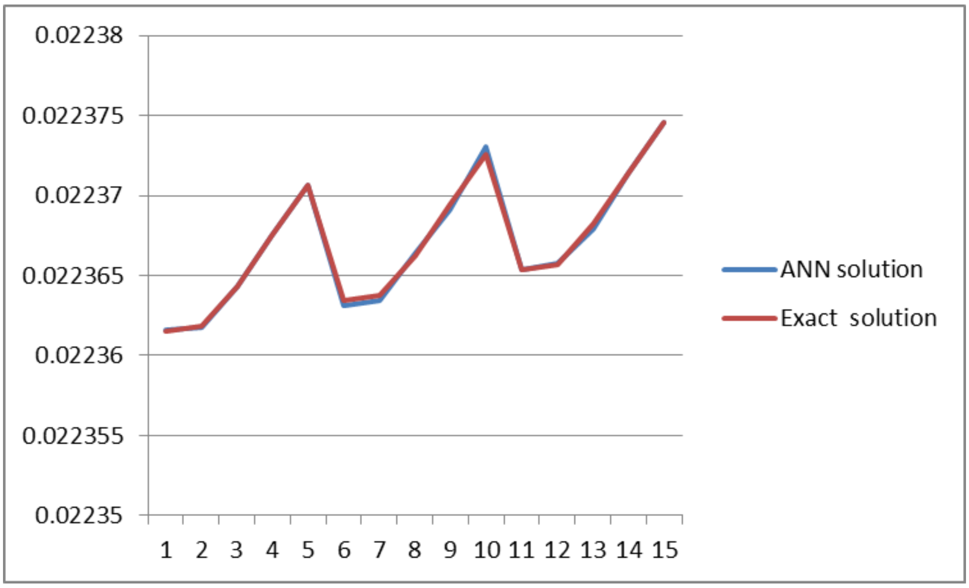

Fig4. Neural Network estimated solution for Example 2(ii)

5. CONCLUSION

We conclude from this paper that artificial neural network, which we proposed, can play an important role to estimate the solutions of the well-known equations; Burgers-Huxley and Burgers-Fisher. It has been shown that the estimated solution by using artificial neural network (ANN) is approximately equal to the exact solution. Although the obtained results have acceptable accuracy, we can improve it by increasing the number of training data, and we can minimize the error and reduce the differences. It is worth mentioning that we used the MATLAB R2011a, to set the algorithm and obtained the data of both solutions.

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